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DYNAMIC PROBLEM OF THE INTERACTION OF A CIRCULAR DIE WITH SOIL REGARDED AS AN ELASTOVISCOPLASTIC HALF-SPACE

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A numerical solution is obtained for an axisymmetric two-dimensional problem concerning the interaction of a circular die with soil regarded as an elastoviscoplastic half-space and subjected to a dynamic load. The problem of the motion of a circular die on an elastic half-space subjected to dynamic loading was solved in [1, 2]. In [3], a two-dimensional formulation was used to numerically solve the problem of the impact of a flat bar-shaped die against a half-space modeling an elastoplastic medium.

In the present study, soil is regarded as an elastoviscoplastic medium with constitutive equations [4] accounting for the effect of strain rate on volume compressibility. Shear strains are described within the framework of an elastoplastic theory of flow [5]. The results are compared with elastic and elastoplastic calculations, as well as with experimental data [6] indicating the need to allow for the viscosity of soil when calculating loads on bodies undergoing dynamic interaction with soil.

1. The deformation of an elastoviscoplastic medium is described by the following system of governing equations [4]:

$$\frac{\partial\varepsilon}{\partial t}=\frac{1}{E}\frac{\partial\sigma}{\partial t}+g(\sigma-f(\varepsilon)),\tag{1.1}$$

$$E=E(\varepsilon),\quad \partial\sigma/\partial t>0;\quad E=E_*(\sigma,\varepsilon),\quad \partial\sigma/\partial t\leqslant 0;$$

$$2G\dot{e}_{ij}=\frac{d\tilde{S}_{ij}}{dt}+\lambda S_{ij},\quad \dot{e}_{ij}=\dot{\varepsilon}_{ij}-\frac{1}{3}\dot{\varepsilon}\delta_{ij},\quad \dot{\varepsilon}=\dot{\varepsilon}_{kk};\tag{1.2}$$

$$J_2=\frac{1}{6}\mathcal{F}^2(\sigma),\quad \sigma=\frac{1}{3}\sigma_{kk}.\tag{1.3}$$

Here, σ_{ij} and ε_{ij} are components of the stress and strain tensors; $J_2 = (1/2)S_{ij}S_{ij}$ is the second invariant of the deviator of the stress tensor; $S_{ij} = \sigma_{ij} - \sigma\delta_{ij}$; $\dot{\varepsilon}_{ij} = \partial\varepsilon_{ij}/\partial t$; $g(z) > 0$ at $z > 0$; $g(z) \equiv 0$ at $z \le 0$; $f(\varepsilon)$ is the statistical compression diagram of the medium; $E(\varepsilon) = \varphi'(\varepsilon)$ is a function characterizing the instantaneous loading of the medium at $\dot{\varepsilon} \to \infty$; $\varphi(\varepsilon)$ is the limiting dynamic compression diagram; G is the shear modulus; $d\tilde{S}_{ij}/dt = dS_{ij}/dt - S_{ik}\Omega_{jk} - S_{jk}\Omega_{ik}$ is the derivative of the stress-tensor deviator in accordance with Jaumann [5].

The plasticity function in (1.3) was written in the form of the following linear relation, in accordance with available empirical data [4]

$$\mathcal{F}(\sigma)=k\sigma+b\tag{1.4}$$

(k and b are empirical coefficients characterizing internal friction and cohesion in the soil).

We now introduce the cylindrical coordinate system i, j, k = x, r, θ. The problem will be solved in an axisymmetric formulation. In this case, the parameters of motion and the stress—strain state of the medium are independent of the angle θ. The x axis coincides with

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the axis of the die. The part of the boundary Γ_1, coincident with the soil surface, is "covered" by a die radius R ($r \le R$) acted upon by a momentary load $P(t) > 0$ at $t \le \tau$ (at $t > \tau$, $P = 0$). The part of the boundary Γ_1 at $r > R$ is free of stresses ($\sigma_{xx} = 0$, $\sigma_{xr} = 0$).

The equation of motion of the die has the form

$$M\frac{dv}{dt} = SP(t) - \int\limits_S \sigma_{xx}\,dS, \tag{1.5}$$

where M is the mass of the die; S is its area; σ_{xx} are the contact stresses under the die.

Besides the boundary Γ_1, in numerically solving the problem we introduce two additional boundaries (nonphysical): a vertical boundary Γ_2 ($r = R_m \ge 3R$) and a horizontal boundary Γ_3 ($x = X_m \ge 0.02\ c_1$, where c_1 is the velocity of elastic longitudinal waves in the soil). These boundaries enclose the theoretical region. It was established that such locations for the boundaries Γ_2 and Γ_3 do not introduce errors into the results calculated for the active phase of the process.

On the boundary Γ_2, we assigned conditions of slip along the rigid surface without friction ($\sigma_{xr}|_{\Gamma_2} = 0$, $v_r|_{\Gamma_2} = 0$, v_r is the radial velocity of the soil). On the boundary Γ_3, we assigned conditions corresponding to contact with a rigid barrier.

2. We used the Wilkins method to numerically solve system (1.1)-(1.5) with the corresponding boundary conditions. The theoretical region was covered by a rectangular grid which was made denser as it approached the die. It was here that the largest stress gradients were found. In the region of contact with the die, $\Delta x = \Delta r = R/8$.

In calculating the stresses in the soil, we used relations that were equivalent to (1.1)-(1.3):

$$dS_{ij} = dS_{ij}^0 - \lambda S_{ij}\,dt, \quad d\sigma = d\sigma^0 - Eg\,dt. \tag{2.1}$$

Here, $dS_{ij}{}^0$ and $d\sigma^0$ are the stress increments obtained with the assumption of triviality of the plastic strains, i.e.,

$$dS_{ij}^0 = (2G\dot{e}_{ij} + S_{ih}\Omega_{jh} + S_{jh}\Omega_{ih}), \quad d\sigma^0 = E\dot{\varepsilon}\,dt. \tag{2.2}$$

Then replacing the differentials by finite increments and allowing for the correspondence between the parameters of the medium and the time layers, in accordance with [7], we find from (2.1) that

$$S_{ij}^{n+1} = S_{ij,n+1}^0 - \lambda^{n+1/2}\Delta t S_{ij}^{n+1}; \tag{2.3}$$

$$\sigma^{n+1} = \sigma_{n+1}^0 - Eg^{n+1/2}\Delta t. \tag{2.4}$$

With allowance for (1.3), we can use (2.3) to obtain an expression similar to that in [7] for $S_{ij}{}^{n+1}$:

$$S_{ij}^{n+1} = S_{ij,n+1}^0 \sqrt{\mathcal{F}(\sigma^{n+1})/(3S_{ij,n+1}^0 S_{ij,n+1}^0)}. \tag{2.5}$$

With allowance for the fact that the shear strain has no effect on the mean stress σ, we calculate the latter for the first step by using the following iteration procedure. The quantity $\Delta\sigma$ is the root of the equation

$$F = g\Delta t + \Delta\sigma/E - \Delta\varepsilon = 0.$$

Considering that the sought value of $\Delta\sigma$ is found in the neighborhood of values $\Delta\sigma_1 = 0$ and $\Delta\sigma_2 = (\Delta f + \Delta\varphi)/2$ (Δf, $\Delta\varphi$ are the stress increments corresponding to the static and dynamic diagrams) and inserting the values of the arguments g and E — corresponding to the middle of the step Δt (with the use of a linear approximation on the interval Δt) — we determine the first approximation

$$\Delta\sigma^{(1)} = (F_2\Delta\sigma_1 - F_1\Delta\sigma_2)/(F_1 - F_2) = -F_1\Delta\sigma_2(F_1 - F_2)$$
$$(F_1 = F(\Delta\sigma_1),\ F_2 = F(\Delta\sigma_2)).$$

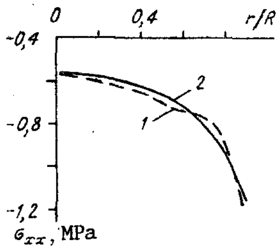

Fig. 1

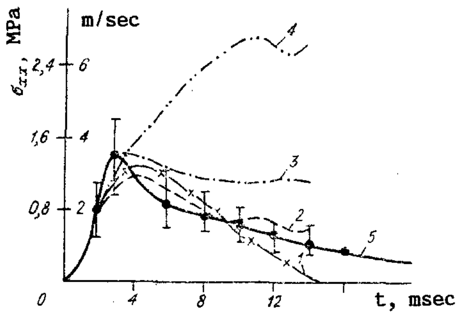

Fig. 2

TABLE 1

Sand	ρ, g/cm³ / w	k / b, MPa	K_1, MPa / E_1, MPa	m / ν	m_1 / ν_1	η_1, $\frac{1}{(\text{MPa})^{\varkappa}\,\text{sec}}$ / $\varkappa$	σ_*, MPa	E_{1*}, MPa / E_{2*}, MPa
Type 1	1,54 / 0	1,60 / 0,04	68,0 / 72,8	610 / 2	100 / 2,24	9,7 / 0,84	8,0	$4\cdot10^3$ / $0{,}5\cdot10^3$
Type 2	1,50 / 0,05	1,65 / 0,04	15,0 / 100,0	840 / 3,4	38 / 2,0	1,9 / 0,5	1,5	$1{,}1\cdot10^3$ / $0{,}17\cdot10^3$

Until the specified accuracy is attained, the quantity $\Delta\sigma^{(j+1)}$ is found in accordance with Newton's scheme

$$\Delta\sigma^{(j+1)} = \Delta\sigma^{(j)} + F(\Delta\sigma^{(j)})/F'(\Delta\sigma^{(j)}) \quad (F' = dF/d(\Delta\sigma)).$$

Calculations were performed for the experimental conditions described in [6], for sandy soils, and for loam and clay [4].

The mechanical characteristics of the soils, entering into Eqs. (1.1)-(1.3), were taken in the form [4]

$$E(\varepsilon) = d\varphi(\varepsilon)/d\varepsilon,\ \varphi(\varepsilon) = E(\varepsilon + m\varepsilon^{\nu}),\ E = E_1/\alpha,$$

$$f(\varepsilon) = K\left(\varepsilon + m_1\varepsilon^{\nu_1}\right),\quad g = \eta(\sigma - f(\varepsilon))^{\varkappa},\quad K = K_1/\alpha,$$

$$\eta = \eta_1\alpha^{\varkappa},\ E_*(\sigma, \varepsilon) = \begin{cases} E_{1*}, & \sigma > \sigma_*, \\ E_{2*}, & \sigma \leqslant \sigma_*, \end{cases}\quad \alpha = 1 + \frac{\sqrt{2}}{3}k.$$

Corresponding values of the empirical coefficients E_1, K_1, and η_1 from [4, 8] are shown in Table 1, where ρ and w are the density of the skeleton and the moisture content of the soil.

Figure 1 compares numerical and analytical (curves 1 and 2) [9] solutions of the given problem in the case of the static introduction of a die with R = 0.6 m into an elastic medium. The results demonstrate the satisfactory accuracy of the given algorithm.

Figure 2 shows results of calculations of the contact stresses under the center of the die (r = 0) with R = 0.3 m and an assigned velocity V(t) on the part of the boundary Γ_1($r \leq R$) (curve 1). This velocity reflects experimental data. Curve 2 shows the same results for an elastoviscoplastic medium [4] (sand of type 1, see Table 1), curve 3 is for the elastoplastic medium, curve 4 for the elastic medium, and curve 5 shows the experimental results from [6] (the vertical lines represent confidence intervals for a confidence level β = 0.95). As the elastoplastic medium, we used Eqs. (1.2), (1.3) with elastic volume deformation.

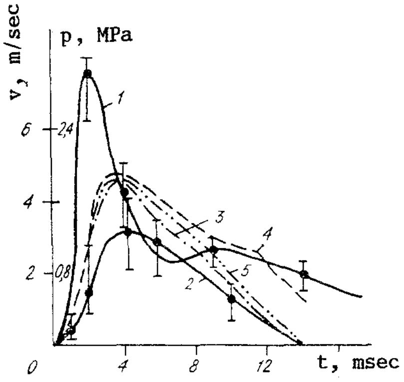

Fig. 3

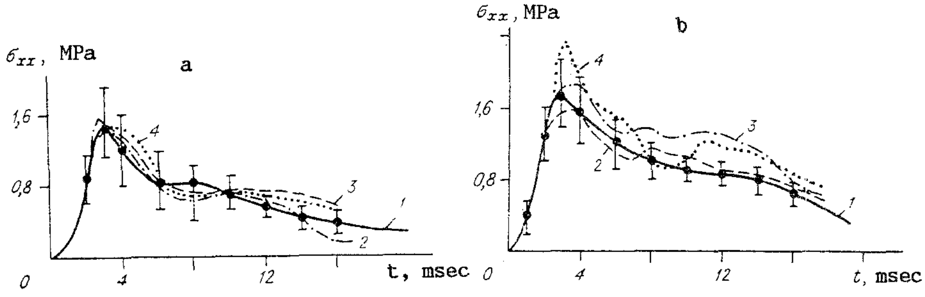

Fig. 4

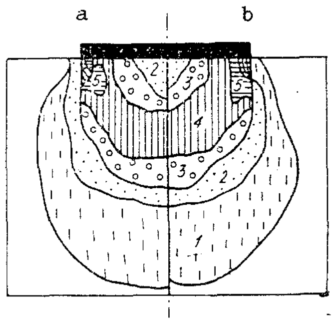

Fig. 5

We should point out the substantial effect of the inelastic properties of the medium on the contact stress — including on the stress averaged over the cross section $\langle\sigma\rangle = \frac{1}{S}\int\limits_S \sigma_{xx}\,dS$.

Here, the best approximation of the test data is obtained with the model which considers the viscoplastic properties of the soil. Results of calculations of the stress in accordance with unidimensional theory for assigned V(t) over the entire boundary Γ_1 agree well with the results of two-dimensional calculations for the stresses under the center of the die at $t < R/c_1$, when the effect of shear is negligible.

A change in the die—soil contact conditions from adhesion to slip without friction led to a 7-12% change in σ_{xx}.

We also performed a series of calculations for the experimental conditions in [6], when the load P(t) on the die was specified in accordance with test measurements. Figures 3 and 4 show the corresponding results of calculations for a die, with R = 0.3 m, located on sandy soil.

Curves 1-5 in Fig. 3 correspond to the load on the die P(t) (line 1) and the velocity of the die v(t) measured in tests and calculated for different conditions: 2 — experimental data from [6]; 3-5 — data calculated on the basis of equations of state (1.1)-(1.3) (2 — sand of type 1; 4 — sand of type 2) and relaxational equations of state from [10] (5 — sand of type 2).

Curves 1-4 in Fig. 4 correspond to the stresses σ_{xx} under the die at the center (a) and on the edge (b): 1 — experimental data; 2, 3 — calculation based on Eqs. (1.1)-(1.3) (2 — sand of type 1; 3 — sand of type 2); 4 — calculation based on the equations in [10] (sand of type 2). It is apparent that die velocity v(t) and the contact stresses $\sigma_{xx}(r, t)$ in its base calculated by means of Eqs. (1.1)-(1.3) agree satisfactorily with the experimental results in [6].

We compared the results of calculation of the velocities v(t) and stresses $\sigma_{xx}(r, t)$ obtained from the model in [10] and the model [4] based on Eqs. (1.1)-(1.3). It was found that the quantitative differences are within the experimental error.

A similar conclusion follows from analysis of data on the stress distribution in sandy soil of type 1 under a dye with R = 0.6. The data are shown in Fig. 5 (a – the model based on Eqs. (1.1)-(1.3); b – the model in [10]). Here, regions 1-5 correspond to the stresses $|\sigma_{xx}| = 0.1n$ MPa, n = 1-5, near the edges of the die $0.6 \le |\sigma_{xx}| \le 0.8$ MPa.

These findings indicate that, at least for the class of problems being examined, the relaxational equation of state proposed in [10] (which is considerably more complex in structure than the model in [4]) does not have any qualitative advantages.

It should be noted that the type of soil (sand, clay, loam) has a significant effect both on the velocity of the die with assigned P(t) on the cover and on the stress state of the soil under the die. We performed calculations for cases similar to those considered in Fig. 3 but with the use of constants characterizing clay and loam – in accordance with [4]. It was found that interaction with these soils is accompanied by a sharp increase in the duration of the phase during which the v(t) decreases after reaching its maximum value. Also, for loam the maximum die velocity increases to 6.6 m/sec (for clay, the velocity nearly coincides with the analogous characteristics for sands).

As was shown by the results of the calculations, the concentration of contact stresses σ_{xx} on the base of the die (Figs. 4 and 5) reaches appreciable values near the edges of the die. This should be taken into account when analyzing experimental data, since the dimensions of the sensors and their location on the base may have a significant effect on the results of measurements.

The theoretical results presented here and their comparisons with empirical data provide evidence of the substantial effect of the viscous properties of soils on parameters of motion of dies and the stress state of soils in their base.

An analysis of the mathematical models in [4, 10] – which account for the viscoplastic properties of soils – showed that, assuming a well substantiated choice of values for the necessary constants, their use in problems involving the interaction of a circular die with soil leads to similar results.

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